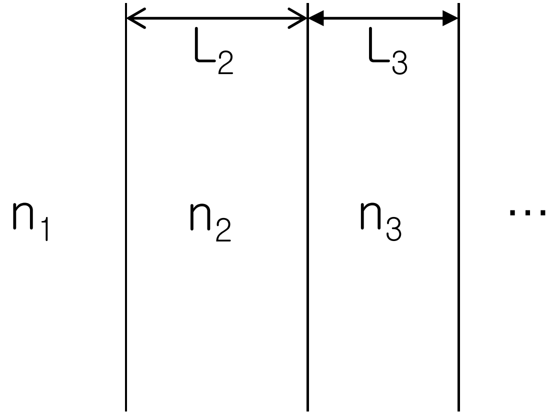
Lect. 7: Multiple Dielectric Interface



Complex Problem:

Requires an advanced technique

Consider two special cases:

1) $L_i = m \frac{\lambda}{2n_i}$ (Half Wavelength)

2) $L_i = (m + \frac{1}{2}) \frac{\lambda}{2n_i}$ (Quarter Wavelength)

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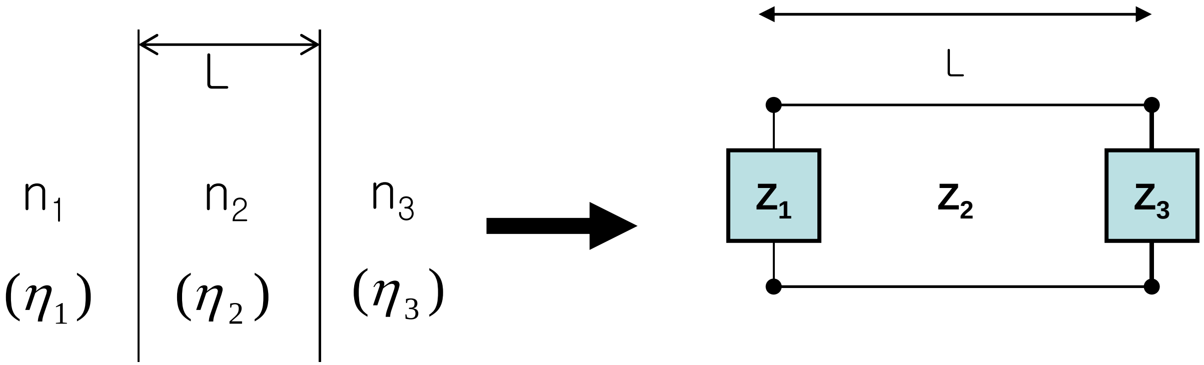
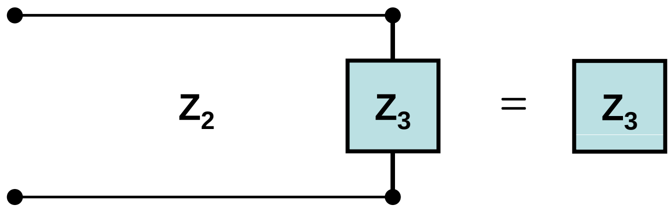
Model dielectric layers with Transmission Lines



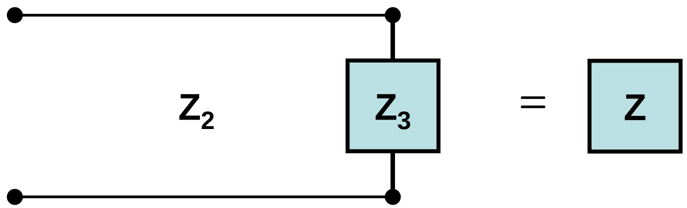
If $L=\frac{\lambda}{2}$

(One rotation on Smith Chart!)



If $L=\frac{\lambda}{4}$

(Half rotation on Smith Chart!)



$$Z=Z_2\frac{1}{Z_3/Z_2}=\frac{Z_2^{\,2}}{Z_3}$$

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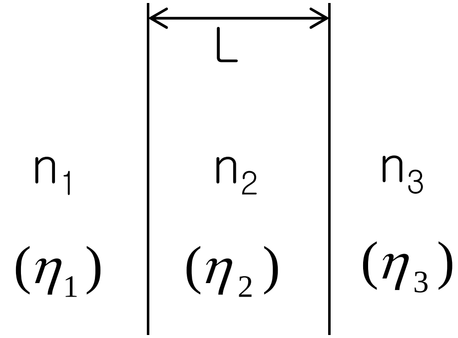

$$L = \frac{\lambda}{2n_2}$$

n_1 | n_3

(η_1) | (η_3)

$$\therefore\ r = \frac{n_1 - n_3}{n_1 + n_3},\quad t = \frac{2n_1}{n_1 + n_3}$$

$$L = \frac{\lambda}{4n_2}$$

n_1 | $n = \frac{n_2^{\,2}}{n_3}$

(η_1) | $(\eta = \frac{\eta_2^{\,2}}{\eta_3})$

$$r = \frac{n_1 - n_2^{\,2}/n_3}{n_1 + n_2^{\,2}/n_3},\quad t = \frac{2n_1}{n_1 + n_2^{\,2}/n_3}$$

$$\therefore\ r = \frac{n_1 n_3 - n_2^{\,2}}{n_1 n_3 + n_2^{\,2}},\quad t = \frac{2n_1 n_3}{n_1 n_3 + n_2^{\,2}}$$

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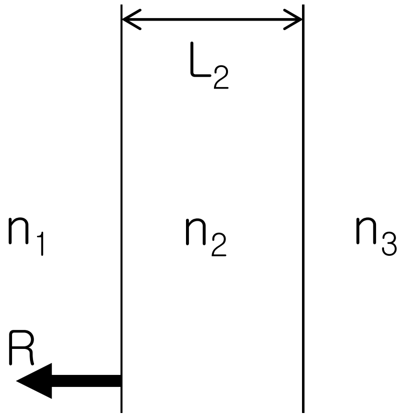
Anti-Reflection coating: Determine L_2 and n_2 so that R=0



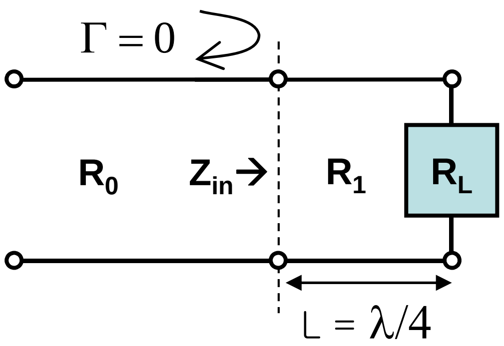
With $L=\dfrac{\lambda}{4n_2}$

n_1 | $n=\dfrac{{n_2}^2}{n_3}$

Since $r=\dfrac{n_1n_3-{n_2}^2}{n_1n_3+{n_2}^2}$,

For r=0, $n_1n_3-{n_2}^2=0$

$$n_2=\sqrt{n_1n_3}$$



$$\therefore R_1=\sqrt{R_0R_L}$$

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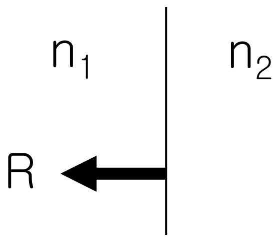
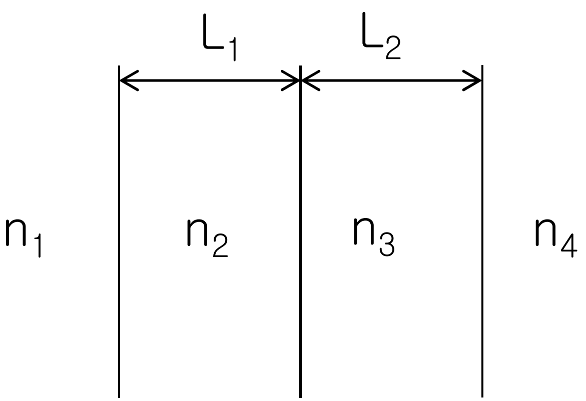
High-Reflection Coating

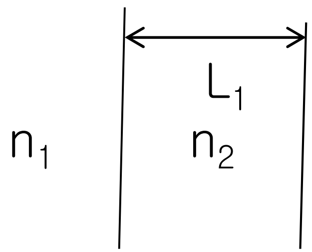


$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \qquad R \to 1 \text{ if } n_1 >> n_2 \text{ or } n_1 << n_2$$

Use quarter-wavelength layers

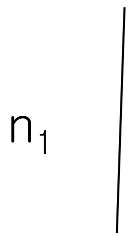


With $L_2 = \frac{\lambda}{4n_3}$



$$n = \frac{n_3^{\ 2}}{n_4}$$

With $L_1 = \frac{\lambda}{4n_2}$



$$\frac{n_2^{\ 2}}{n_3^{\ 2} / n_4}$$

$$R = \left(\frac{n_1 - \left(\frac{n_2}{n_3}\right)^2 n_4}{n_1 + \left(\frac{n_2}{n_3}\right)^2 n_4} \right)^2$$

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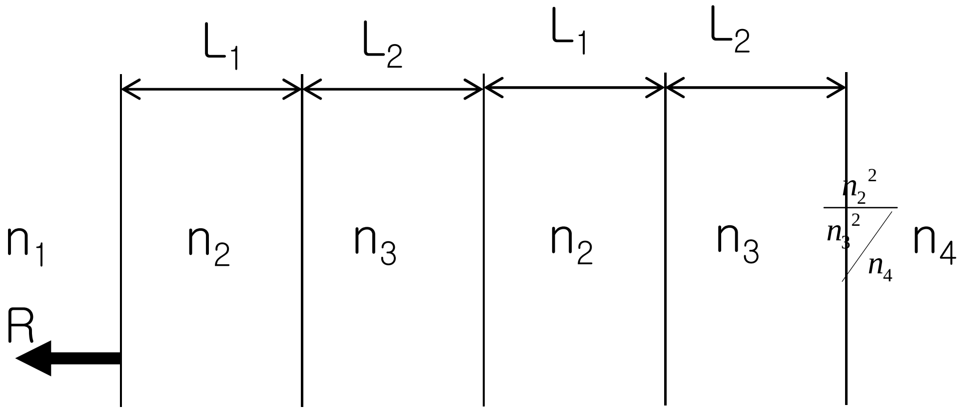
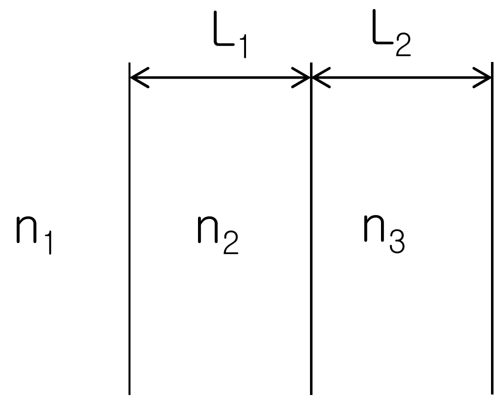
High-Reflection Coating

Repeat the quarter-wavelength pair





$$\frac{n_2^{\,2}}{n_3^{\,2}/n_4}$$

Substitute $\dfrac{n_2^{\,2}}{n_3^{\,2}/n_4}$ for n_4 in $R = \left(\dfrac{n_1 - \left(\dfrac{n_2}{n_3}\right)^2 n_4}{n_1 + \left(\dfrac{n_2}{n_3}\right)^2 n_4} \right)^2$

Then, $R = \left(\dfrac{n_1 - \left(\dfrac{n_2}{n_3}\right)^4 n_4}{n_1 + \left(\dfrac{n_2}{n_3}\right)^4 n_4} \right)^2$

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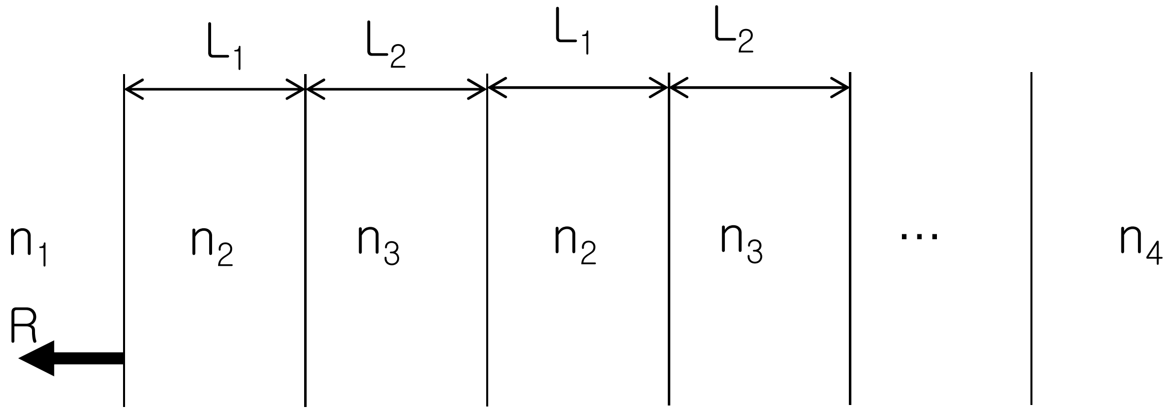
High-Reflection Coating

Repeat the quarter-wavelength pair m times.



$$R = \left(\frac{n_1 - \left(\frac{n_2}{n_3}\right)^{2m} n_4}{n_1 + \left(\frac{n_2}{n_3}\right)^{2m} n_4} \right)^2$$

$$\text{If } n_2 > n_3,\ R \sim \left(\frac{-\left(n_2/n_3\right)^{2m} n_4}{+\left(n_2/n_3\right)^{2m} n_4} \right)^2 = 1$$

$$\text{If } n_2 < n_3, R \sim \left(\frac{n_1}{n_1} \right)^2 = 1$$

➔ Dielectric mirror

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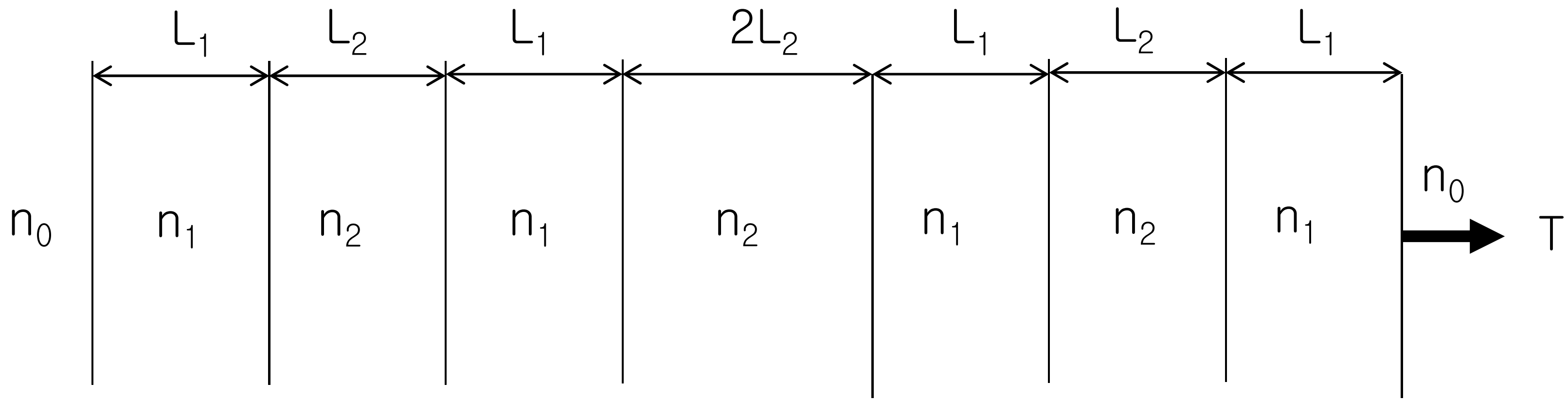
Homework: Due on 9/27 before Tutorial

Determine T for the following multiple dielectric layers.



$$n_1 = 1.35,\ n_2 = 2.3,\ L_{1,2} = \frac{\lambda}{4n_{1,2}}$$